Abstract

The 2D passive earth pressures acting on rigid retaining walls problem has been widely treated in the literature using different approaches (limit equilibrium, limit analysis, slip line and numerical computation), however, the 3D passive earth pressures problem has received less attention. This paper is concerned with the numerical study of 3D passive earth pressures induced by the translation of a rigid rough retaining wall for associated and non-associated soils. Using the explicit finite difference code FLAC3D, the increase of the passive earth pressures due to the decrease of the wall breadth is investigated. The results given by the present numerical analysis are compared with other investigation. The influence of the angle of dilation on the coefficients is also studied.

Key words: Numerical modeling, FLAC 3D, retaining wall, passive earth pressures, angle of dilation

1. Introduction

The determination of passive earth pressure acting on rigid retaining structures is a classical geomechanical problem. Safe and economical design of retaining walls requires a sound knowledge of the contact pressure exerted against. In spite of more sophisticated methods have been developed in recent decades, the classical earth pressure theories of Coulomb[1], Rankine[2] and Caquot-Kérisel [3] still occupy a dominant place in geotechnical engineering practice.

The 2D passive earth pressures problem has been widely treated in the literature theoretically using different approaches (limit equilibrium, limit analysis, slip line and numerical computation), however, the 3D passive earth pressures problem has been received theoretically less attention apart from the work of Blum [4] using the limit equilibriunm approach and Soubra and Regenass [5] using the upper bound theorem of limit analysis. This is indeed due to the difficulty to propose analytically the 3D failure mechanisms.

Dilatancy is the change of volume that occurs when a soil is sheared. It is the manifestation of entanglement or disentanglement between particles when applied to the soil shear forces. For soils, experimental observations have shown that the frictional soil is not associated material characterized by a dilatancy angle $\psi$ often much lower than the friction angle $\phi$. 

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To take into account the effect of the nonassociativity, some authors (Drescher and Detournay [6]; Michalowski and Shi [7]) suggest modifying the values of cohesion \( c \) and friction angle \( \phi \) by \( c^* \) and \( \phi^* \).

In this research, 3D earth pressure analyses have been performed using the explicit finite difference code, Fast Lagrangian Analyses of Continua FLAC3D [8]. The aim of this work is firstly to develop a numerical methodology for the analysis of a rigid rough retaining wall subjected to translation, then to investigate the influence of dilatancy angle \( \psi \) on the evaluation of the earth pressure coefficients. An interpretation and discussion of the numerical results obtained from the present analysis conclude this paper.

2. Numerical Modeling Procedure

Numerical simulation of passive earth pressure acting on rigid rough vertical retaining wall (as shown in figure 1) is concerned in this paper using the commercially available three-dimensional code FLAC3D [8]. This code uses an explicit finite difference program to study numerically the mechanical behaviour of a continuous 3D medium as it reaches equilibrium or steady plastic flow. The explicit Lagrangian calculation scheme and the mixed-discretization zoning technique (Cundall PA.[9]) used in FLAC3D ensure that plastic failure and flow are modelled very accurately (Benmebarek et al. [10]).

For the soil behavior, a linear elastic-perfectly plastic Mohr–Coulomb model encoded in FLAC3D is adopted. This robust model widely known and used in the simulations of geotechnical structures was selected and it has the advantage of requiring few parameters whose meaning is well represented, requiring the specification of a shear modulus \( G = 60 \text{ MPa} \), a bulk modulus \( K = 22 \text{ MPa} \), a unit weight \( \gamma = 20 \text{ kN/m}^3 \), a friction angle \( \phi \), and an angle of dilation \( \psi \) [10]. It is noted that passive and active earth pressure coefficients are independent from elastic and weight soil parameters. In the code FLAC3D, it is preferable for the elastic properties of geomaterials to use the bulk modulus \( K \) and shear modulus \( G \) as the Young's modulus \( E \) and Poisson's ratio \( \nu \).
The interface model incorporated in FLAC3D code and its components illustrated in Fig. 2 have been used to simulate the soil/wall contact. For this problem, half-symmetry condition is assumed in the numerical simulation. The domain used for the analysis is sketched in Fig. 3, together with its dimensions and axis[10]. Fig 4 shows an example of the mesh retained for this analysis. The grid size is fine near the wall where deformations are concentrated. The proposed modeling procedure of the passive earth pressures are based on two steps:

In the first one, the geostatic stresses are computed. At this stage some stepping is required to bring the model to equilibrium.

In the second step, to represent rigid wall translation, a controlled horizontal velocity is applied in several steps to all the wall grid points.

Hence, the value of the passive earth pressure force $P_p$ (1,2) can be deduced from the following relationship:

$$P_p = \frac{P_{px}}{\cos \delta}$$  \hspace{1cm} (1)

$$P_p = K_{p\gamma(3D)} \gamma \frac{h^2}{2} b$$ \hspace{1cm} (2)

Where $P_p$ is the passive earth force; $P_{px}$ is the horizontal passive earth force, $\gamma$ is the unit weight of the soil; $h$ is the penetration depth of the wall; $b$ is the breadth of the wall; $\delta$ is the soil-wall interface friction angle; $K_{p\gamma(3D)}$ is the 3D passive earth pressure coefficient (represent the influence of soil weight, cohesion $c=0$).

Following the first and the second steps of the proposed modeling procedure, the $K_{p\gamma(3D)}$ coefficient is determined from the following expression:

$$K_{p\gamma(3D)} = 2P_{px} / \gamma \cdot h^2 \cdot b \cdot \cos \delta$$ \hspace{1cm} (3)
3. Overview of Previous Work

To investigate how the passive earth pressure coefficients are affected by the wall breadth b/h=0.5 and b/h=10, two values of the internal friction angle $\phi$ 25° and 35° are considered for the value $\delta/\phi=1$ of soil-wall interface friction. The computed values of passive earth pressure coefficients $K_p\gamma(3D)$ are listed in Tables 1 and 2 and compared to solutions given by Soubra and Regenass [5], Skrabl, S and Macuh, B. [11], Helena and al. [12]. These tables clearly show the sensitivity of the passive earth pressure coefficients to the b/h ratio.

**Table 1**: comparison of $K_p\gamma(3D)$ coefficients for $b/h=0.5$ ; $\delta/\phi=1$

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$\phi =25^\circ$</td>
<td>12.776</td>
<td>10.985</td>
<td>10.431</td>
<td>7.913</td>
</tr>
<tr>
<td>$\phi =35^\circ$</td>
<td>54.064</td>
<td>40.135</td>
<td>37.509</td>
<td>23.645</td>
</tr>
</tbody>
</table>

For large values of b/h, the 3D effect vanishes and the $K_p\gamma(3D)$ converge to the 2D coefficients. For instance for the present results (with Flac3D), $b/h=10$ compared to the 2D results of Caquot and Kérisel [3] we have:

For $\phi=25^\circ$, $\delta/\phi=1$, $K_p\gamma(3D) =4.412$ and $K_p\gamma(2D) =4.40$;  
For $\phi=35^\circ$, $\delta/\phi=1$, $K_p\gamma(3D) =10.675$ and $K_p\gamma(2D) =10.50$;

It is interesting to note that $K_p\gamma(3D)$ coefficients (for the present simulation) for large values of b/h ($b/h=10$) are near to $K_p\gamma(2D)$ than the other solutions on table 2.

**Table 2**: comparison of $K_p\gamma(3D)$ coefficients for $b/h=10$ ; $\delta/\phi=1$

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi =25^\circ$</td>
<td>5.004</td>
<td>4.885</td>
<td>4.579</td>
<td>4.412</td>
</tr>
<tr>
<td>$\phi =35^\circ$</td>
<td>13.730</td>
<td>12.857</td>
<td>12.131</td>
<td>10.675</td>
</tr>
</tbody>
</table>

To investigate how the passive earth pressure coefficients are affected by the soil dilation angle, Tables 4 give the passive earth pressure coefficients $K_p\gamma(2D)$ for different results Soubra[13], Benmebarek and al[14] and Caquot[3]. Four values of the angle of internal friction $\phi=20^\circ$, $30^\circ$, $35^\circ$, $40^\circ$, three values of the friction angle at the soil-wall interface $\delta/\phi=0$, $1/3$, $2/3$ and three values of the dilation angle $\psi/\phi=0$, $1/2$, $1$ are considered in the analysis.
From these results (Table 4), it is noted that the soil dilatancy angle $\psi$ has a remarkable effect on the earth passive pressure coefficient only for soil presenting high friction ($\phi > 30^\circ$), whereas its influence is practically negligible for $\phi = 20^\circ, 30^\circ$.

The results of simulation with Flac 2D [15] frame well the values proposed by Caquot and Kérisel[3] with a difference not exceeding 7% for $\psi/\phi = 1$.

Table 3 gives some values of $K_p(2D)$ coefficients obtained whereas in the one hand associated material $\psi=\phi$ and secondly a non-associated material with $\psi=0.25\phi, 0.5\phi, 0.75\phi$. The results shows an increase of the active pressure coefficient $K_p(2D)$ with the increase of soil dilatancy angle particularly for high friction angle, this values proposed by Regenass[16]

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\psi = 0.25\phi$</th>
<th>$\psi = 0.50\phi$</th>
<th>$\psi = 0.75\phi$</th>
<th>$\psi = \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2.21</td>
<td>2.23</td>
<td>2.24</td>
<td>2.25</td>
</tr>
<tr>
<td>20</td>
<td>2.98</td>
<td>3.06</td>
<td>3.10</td>
<td>3.12</td>
</tr>
<tr>
<td>25</td>
<td>4.10</td>
<td>4.31</td>
<td>4.46</td>
<td>4.51</td>
</tr>
<tr>
<td>30</td>
<td>5.73</td>
<td>6.29</td>
<td>6.70</td>
<td>6.86</td>
</tr>
<tr>
<td>35</td>
<td>8.14</td>
<td>9.52</td>
<td>10.66</td>
<td>11.13</td>
</tr>
<tr>
<td>40</td>
<td>11.70</td>
<td>15.02</td>
<td>18.15</td>
<td>19.62</td>
</tr>
<tr>
<td>45</td>
<td>17.01</td>
<td>24.83</td>
<td>33.75</td>
<td>38.61</td>
</tr>
</tbody>
</table>

Table 4. variation of $K_p(2D)$ coefficients according to the dilatancy angle

<table>
<thead>
<tr>
<th>$\phi$ (°)</th>
<th>$\psi/\phi$</th>
<th>$\psi = \phi$</th>
<th>$\psi = 0.25\phi$</th>
<th>$\psi = 0.50\phi$</th>
<th>$\psi = 0.75\phi$</th>
<th>$\psi = \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0</td>
<td>2.07</td>
<td>2.04</td>
<td>2.04</td>
<td>2.38</td>
<td>2.77</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2.08</td>
<td>2.39</td>
<td>2.39</td>
<td>2.77</td>
<td>2.77</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>2.99</td>
<td>3.00</td>
<td>3.00</td>
<td>4.03</td>
<td>4.03</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.09</td>
<td>4.05</td>
<td>4.05</td>
<td>5.21</td>
<td>5.21</td>
</tr>
<tr>
<td>35</td>
<td>0</td>
<td>3.53</td>
<td>5.06</td>
<td>5.06</td>
<td>7.05</td>
<td>7.05</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.79</td>
<td>5.46</td>
<td>5.46</td>
<td>7.60</td>
<td>7.60</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>4.02</td>
<td>3.69</td>
<td>3.69</td>
<td>6.37</td>
<td>6.37</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4.61</td>
<td>7.54</td>
<td>7.54</td>
<td>11.63</td>
<td>11.63</td>
</tr>
</tbody>
</table>

4. Results and Discussion

To investigate how the 3D passive earth pressure coefficients are affected by the soil dilatation
angle we presented three cases:

**First case: smooth and slender wall** \((\delta=0, \frac{b}{h}=0.1)\) for dilatant \((\phi=\psi=20^\circ)\) and nondilatant soil \((\frac{\psi}{\phi}=1/2, \phi=20^\circ, \psi=10^\circ)\) the coefficient has the same value, \(K_p\gamma(3D) = 6.51\)

For \(\delta=0, \frac{b}{h}=0.25\) and \(\phi=\psi=20^\circ\) (dilatant soil) \(K_p\gamma(3D) = 4.55\) while for the same case but \(\psi=15^\circ\) (nondilatant soil) \(K_p\gamma(3D) = 4.51\)

**Second case: rough and slightly slender wall** \((\delta=\phi=40^\circ, \frac{b}{h}=0.5)\) for dilatant soil \((\phi=\psi=40^\circ)\) \(K_p\gamma(3D) = 45.46\) for the same case but for nondilatant soil \((\psi=10^\circ)\) \(K_p\gamma(3D) = 34.91\)

**Third case: influence of the roughness and the friction angle** \((\frac{b}{h}=2, \phi=40^\circ)\)

<table>
<thead>
<tr>
<th>Roughness of the wall</th>
<th>Dilatant soil (\psi=40^\circ)</th>
<th>Nondilatant soil (\psi=20^\circ)</th>
<th>Nondilatant soil (\psi=0^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth wall (\delta=0^\circ, \phi=40^\circ)</td>
<td>7.02</td>
<td>6.94</td>
<td>6.26</td>
</tr>
<tr>
<td>Rough wall (\delta=\phi=40^\circ)</td>
<td>24.90</td>
<td>23.43</td>
<td>15.28</td>
</tr>
</tbody>
</table>

In order to study the influence of dilation angle on the value of three-dimensional (3D) earth pressure coefficients \(K_p\gamma(3D)\) several cases are considered in the analysis:

- Influence of the wall dimension \(b\times h\) \((b=\text{breadth}; h=\text{height})\) \(b/h=0.1, b/h=0.25, b/h=0.5, b/h=2,\)
- Influence of friction angle \(\phi=20^\circ\) et \(40^\circ\),
- Influence of soil-wall interface friction angle \(\delta=0^\circ\) et \(\delta=\phi,\)

From these results, it is noted that the 3D earth pressure coefficient \(K_p\gamma(3D)\) decreases with decreasing angle of dilatancy.

The results show the influence of friction angle \(\phi\), taking into account the nondilatant nature of the soil is more pronounced than the value of \(\phi\) is important

Table 5 shows an increase of \(K_p\gamma(3D)\) coefficient with increase soil-wall interface friction angle \(\delta\) and soil dilatancy angle \(\psi\) has a remarkable effect for rough wall, for high friction angle the difference reaches 61% between dilatant and nondilatant soil.

the wall dimension \(b\times h\) have influence on dilatancy, the higher the ratio \(b/h\) is great plus the influence of the dilatancy angle is remarkable on \(K_p\gamma(3D)\).

Figure 5 and 6 show respectively for two values of the dilatancy angle \((\psi=\phi=40^\circ, \psi=0^\circ)\) the displacement field vectors and 3D failure mechanism for dilatant (a) and nondilatant (b) soil. The distance of the surface failure in plan view from the front of the wall is in congruous with experimental observations. These surfaces are smooth as observed experimentally by Meksaouine[17],
5. Conclusions

Numerical computations of the passive earth pressure coefficients have been performed using FLAC3D code. In conclusion, the dilatancy has an important influence on the passive earth pressure coefficients, the solutions presented are given for associative and non-associative material. A number of conclusions may be drawn from this investigation:

- The passive earth pressures coefficients decrease with a decrease in the dilation angle $\psi$ for great $\phi$ values (more than 30°). For instance the difference reaches 61% between dilatant and nondilatant soil for high friction angle ($\phi=40°$).
- The wall dimension $b/h$ have influence on dilatancy, the higher the ratio $b/h$ is great plus the influence of the dilatancy angle is remarkable on $Kp_\gamma(3D)$.
- The 3D (present work) and 2D [14,16], passive earth pressure coefficients decreases with the dilation angle decrease particularly when $\psi/\phi$ varied from 1/2 to 0.
- The roughness of the wall ($\delta/\phi$) has a significant influence on the dilatancy, the wall is more rough over the influence of dilatancy is important.
- The distance of the surface failure in plan view from the front of the wall is in congruous with experimental observations.
- The failure surface of a nondilatant soil is less important than a dilatant soil.
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