ACTIVE CONTROL OF A QUARTER-CAR MODEL SUSPENSION SYSTEM WITH SLIDING MODE AND PID CONTROL APPROACHES

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Abstract

This study covers a State Dependent Ricatti Equation (SDRE) based Sliding Mode Controller (SMC) and classical PID controller that are applied for an active control of the suspension system of a quarter car model. At first, this suspension system is controlled by SDRE based sliding mode controller for a road disturbance input and then for the same input the same system is controlled by classical PID controller. Finally, the results are compared.

Key Words: Suspension systems, SDRE Based Sliding Mode Control, PID controller

1. Introduction

Conventional vehicle suspension systems are used for isolation of road-induced vibration. There are three common types of suspension systems. These are active, semiactive and passive suspensions. Passive suspension system means that it has springs, dampers or shock absorbers. On the other hand, the semi active suspension systems have controllable dampers as electrorheological (ER) and magnetorheological (MR). Both fluids are smart materials made mixing fine particles into a liquid with low viscosity. Active suspensions mean that at least some part of required suspension force is generated from active power sources such as compressors, hydraulic pumps, etc. In application of active suspensions, microprocessors, associated electronic devices and also actuators are used. Compared with passive suspensions, active suspensions can improve the performance of the suspension system over a wide range of frequency[1-3].

The present work includes active control of an quarter car model subjected to random road disturbance input. For control of active power source, both SDRE based sliding mode and classical PID controllers are used. The main purpose of this study is to consume less energy by generating less force while the road-induced vibrations are isolating.

The mathematical backround of controllers are given in Section 2 of the paper. In Section 3, mathematical model of car and suspension system are given. The simulation results are given in Section 4 and finally conclusions are in Section 5.

2. Materials and Method

2.1 Theory

In this study, two types of control techniques are used. The first technique is SDRE based SMC and the second technique is classical PID controller.

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The system to be controlled is described as follows.
\[
\dot{x}(t) = Ax(t) + Bu(t) \tag{2.1}
\]
Where \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \) are state and control vectors. Since the system is considered to be frozen at each time interval, the system is approached as a LTI system at that time interval. Therefore, the system becomes as follows.
\[
\dot{x}(t) = Ax(t) + Bu(t) \tag{2.2}
\]
Then if the coordinate transformation is used on that system as follows.
\[
z(t) = T_r x(t) \tag{2.3}
\]
\[
T_r = q(B) \tag{2.4}
\]
Where \( q(B) \) is Q-R decomposition matrix of matrix B. And then the system and control matrices in terms of that decomposition can be written as follows.
\[
A_{\text{reg}} = T_r A T_r^T \tag{2.5}
\]
\[
B_{\text{reg}} = T_r B \tag{2.6}
\]
For SDRE algorithm, a performance index must be determined. That cost function is given as
\[
J = \frac{1}{2} \int_t^{\infty} x(t)^T Q x(t) dt \tag{2.7}
\]
where \( Q \in \mathbb{R}^{nxn} \), symmetric and positive definite matrix. In addition, \( t_s \) is the time when the system begins to sliding motion. At that time \( x(t) \to 0 \). After that if the coordinate transformation above is applied on the matrix \( Q \).
\[
Q_{\text{reg}} = T_r Q T_r^T \tag{2.8}
\]
Matrices in Equation (2.4), Equation (2.5) and Equation (2.8) are defined as in [4]
\[
A_{\text{reg}} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{2.9}
\]
\[
B_{\text{reg}} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{2.10}
\]
Where \( A_{11} \in \mathbb{R}^{(n-n-1)x(n-m-1)} \), \( A_{12} \in \mathbb{R}^{(n-m-1)x(m-1)} \), \( A_{21} \in \mathbb{R}^{(m-1)x(n-m-1)} \), \( A_{22} \in \mathbb{R}^{(m-1)x(m-1)} \) and \( B_2 \in \mathbb{R}^{(m-1)x(m-1)} \).
\[
T_r Q T_r^T = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \tag{2.11}
\]
Then if the variables in Equation (2.3) and Equation (2.11) is introduced into Equation (2.7), the performance index is defined as
\[
J = \frac{1}{2} \int_t^{\infty} z_1^T Q_{11} z_1 + 2 z_1^T Q_{12} z_2 + z_2^T Q_{22} z_2 dt \tag{2.12}
\]
But performance index in Equation (2.12) is not in standard LQR form. Because of this the variable \( 2z_1^T Q_{12} z_2 \) in Equation (2.12) must be eliminated. For elimination the steps given below should be followed.
\[
2z_1^T Q_{12} z_2 + z_2^T Q_{22} z_2 = (z_2 + Q_{21} Q_{22}^{-1} Q_{12} z_1)^T . Q_{22} (z_2 + Q_{21} Q_{22}^{-1} Q_{12} z_1) - z_1^T Q_{12} Q_{22}^{-1} Q_{12} z_1 \tag{2.13}
\]
Then, if new variables are defined as follows.
\[
\bar{Q} = Q_{11} - Q_{12} Q_{22}^{-1} Q_{21} \tag{2.14}
\]
And
\[
\nu = z_2 + Q_{22}^{-1} Q_{21} z_1 \tag{2.15}
\]
The new cost function is obtained after the performance index in Equation (2.12) is redefined with the new variables in Equation (2.13), Equation (2.14) and Equation (2.15) as
\[
J = \frac{1}{2} \int_t^{\infty} (z_1^T \bar{Q} z_1 + \nu^T Q_{22} \nu) dt \tag{2.16}
\]
Therefore, performance index in Equation (2.16) have transformed into its regular form. Then if we rewrite coordinate transformation as
\[
\bar{z}_1(t) = A_{11} z_1(t) + A_{12} z_2(t) \tag{2.17}
\]
And if a new parameter is defined as below
\[
\bar{A} = A_{11} - A_{12} Q_{22}^{-1} Q_{21} \tag{2.18}
\]
Then if the parameters in Equation (2.15) and Equation (2.18) are inserted into Equation (2.17) the result is as followings.

$$\dot{z}_1(t) = \hat{A}z_1(t) + A_{12}v(t)$$  \hspace{1cm} (2.19)

If the performance index in equation (2.16) is solved by using state dependent Riccati equation the solution is obtained as

$$P_1 \hat{A} + \hat{A}^T P_1 - A_{12} Q_{22} A_{12}^T P_1 + \bar{Q} = 0$$  \hspace{1cm} (2.20)

In equation (2.20) $P_1$ is positive definite and solution of the Riccati equation. Then if an optimal $v$ is defined as

$$v = -Q_{22}^{-1} A_{12}^T P_1 z_1$$  \hspace{1cm} (2.21)

Then if this optimal term is equalized with the term in equation (2.15) and after that if new equation is solved the result is given below.

$$z_2 = -Q_{22}^{-1} (A_{12}^T P_1 + Q_{21}) z_1$$  \hspace{1cm} (2.22)

Finally from the equation (2.22), the sliding surface’s slope is obtained as follows.

$$M = -Q_{22}^{-1} (A_{12}^T P_1 + Q_{21})$$  \hspace{1cm} (2.23)

In equation (2.23) $M \in \mathbb{R}^{n \times m}$. After finding the slope, the sliding surface is obtained as

$$\sigma(z) = z_2 + M z_1$$  \hspace{1cm} (2.24)

To produce further, a control term which hold the system on designed sliding surface should be defined. For this reason, the condition below must be satisfied [4].

For $\sigma(z) > 0$ \hspace{0.2cm} $\dot{\sigma}(z) < 0$, for $\sigma(z) < 0$ \hspace{0.2cm} $\dot{\sigma}(z) > 0$

Here, $\dot{\sigma}(z)$ is the first time derivative of the sliding surface.

$$\dot{\sigma}(z) = M A z$$  \hspace{1cm} (2.26)

The control term is divided into two parts as follows.

$$u = u_{eq} + u_{non}$$

$$u_{eq} = - (\sigma B)^{-1} \dot{\sigma}$$

$$u_{non} = -k(t, z)(\sigma B)^{-1} \text{sgn}(\sigma)$$  \hspace{1cm} (2.29)

Here, $k(t, z) > 0$. Before advancing to the next step, it is first pertinent to establish sufficient conditions which guarantee that an ideal sliding motion will take place. Intuitively, the sliding surface must be at least locally attractive. This may be expressed mathematically as

$$\lim_{\sigma \to 0^+} \dot{\sigma} < 0 \quad \text{and} \quad \lim_{\sigma \to 0^-} \dot{\sigma} > 0$$  \hspace{1cm} (2.30)

In some domain $\Omega \subset \mathbb{R}^n$. In this case the sliding surface would be

$$D = \sigma \cap \Omega = \{ \mathbf{x} \in \Omega : \sigma(\mathbf{x}) = 0 \}$$  \hspace{1cm} (2.31)

The expression given in equation (2.30) is often replaced by the equivalent, but more succinct criterion [4]

$$\begin{align*}
\sigma \dot{\sigma} &< 0 \\
\frac{1}{2} \frac{d}{dt} \sigma^2 &\leq \sigma \dot{\sigma}
\end{align*}$$  \hspace{1cm} (2.33)

It follows that the function

$$V(\sigma) = \frac{1}{2} \sigma^2$$  \hspace{1cm} (2.34)

is a Lyapunov function for the state $\sigma$. Unfortunately, although equations (2.30) and (2.32) are commonly encountered in the literature, they do not guarantee the existence of an ideal sliding motion [4]. Essentially these conditions only guarantee that the sliding surface is reached asymptotically.

A stronger condition, guaranteeing an ideal sliding motion, is the $\eta$-reachability condition given by [4]

$$\dot{\sigma} \sigma \leq -\eta |\sigma|$$  \hspace{1cm} (2.35)

Here, $\eta$ is a small positive constant. By rewriting equation (2.33) a
\[ \frac{1}{2} \frac{d}{dt} \sigma^2 \leq -\eta |\sigma| \]  

(2.36)

and integrating from 0 to \( t_s \), it follows that

\[ |\sigma(t_s)| - |\sigma(0)| \leq -\eta t_s \]  

(2.37)

and thus the time taken to reach \( \sigma = 0 \), represented by \( t_s \), satisfies condition below[4,5].

\[ t_s \leq \frac{|\sigma(0)|}{\eta} \]  

(2.38)

and for PID control

\[ u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \]  

(2.39)

where \( K_p, K_i, K_d \) and \( e(t) \) are proportional, integral, derivative gain constants and error function respectively[6].

3. Dynamics of Car Model

In this study, a simple quarter-car suspension model that consists of one-fourth of the body mass, suspension components and one wheel is shown in Figure 1. This model has been used extensively in the literature and captures many essential characteristics of a real suspension system. The equations of motion and figure are borrowed from [7].

\[
\begin{align*}
    m_s \ddot{z}_s + c_s [\dot{z}_s(t) - \dot{z}_u(t)] + k_s [z_s(t) - z_u(t)] &= -u(t) \quad (3.1) \\
    m_u \ddot{z}_u + c_s [\dot{z}_u(t) - \dot{z}_s(t)] + k_s [z_u(t) - z_s(t)] + k_t [z_u(t) - z_r(t)] &= u(t) \quad (3.2)
\end{align*}
\]

![Quarter-car model](image)

Figure 1. Quarter-car model [7]

where \( m_s \) is the sprung mass, which represents the car chassis; \( m_u \) is the unsprung mass, which represents the wheel assembly; \( c_s \) and \( k_s \) are damping and stiffness of the uncontrolled suspension system, respectively; \( k_t \) serves to model the compressibility of the pneumatic tyre; \( z_s \) and \( z_u \) are the displacements of the sprung and unsprung masses, respectively; \( z_r \) is the road displacement input; \( u(t) \) represents the external input force of the suspension system.

For sliding mode control Equation (3.1) and (3.2) must be transform to form below,

\[ \dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \]  

(3.3)

where
\( A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ -k_s/m_u & 0 & -c_s/m_s & c_s/m_s \\ k_s/m_s & -k_t/m_s & c_s/m_s & -c_s/m_s \end{bmatrix} \) (3.4)

\( B = \begin{bmatrix} 0 \\ 0 \\ -1/m_s \\ 1/m_u \end{bmatrix} \) (3.5)

\( B_w = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \) (3.6)

After choosing the states as
\[
\begin{align*}
    x_1(t) &= z_s(t) - z_u(t) \\
    x_2(t) &= z_u(t) - z_r(t) \\
    x_3(t) &= \dot{z}_s(t) \\
    x_4(t) &= \dot{z}_u(t)
\end{align*}
\]

where \( x_1(t) \) represents suspension deflection, \( x_2(t) \) is the tyre deflection, \( x_3(t) \) is the sprung mass speed, \( x_4(t) \) is unsprung mass speed and \( w(t) \) denotes the disturbance caused by road roughness.

And for PID control, the analyzes must be done in s domain. Therefore, the transfer functions between inputs and outputs must be determined. In that analyze \( x_s \) is used as output and \( u(t) \) and \( d(t) \) are used as input. So for defining the first transfer function between \( x_s \) and \( d(t) \) the laplace transform of Eq(3.1) and (3.2) must be taken as follows,
\[
0 = m_s s^2 X_s + k_s X_s - k_s X_u + c_s s X_s - c_s s X_u
\]

From Eq(8)
\[
X_u = X_s \left( \frac{m_s s^2 + c_s s + k_s}{c_s + k_s} \right)
\]

Then if the laplace transform of Eq(3.2) is taken and rearrange with Eq(3.13), the first transfer function between \( x_c \) and \( w \) will be like that,
\[
Tf_1 = \frac{c_s k_t s + k_s k_t}{\Delta}
\]

Where
\[
\Delta = m_u m_s s^4 + s^3 (m_c s + m_u c_s) + s^2 (m_s k_t + m_s k_1 + m_u k_1) + c_s k_t s + k_s k_t
\]

And if the same calculations is done the second transfer function between \( x_s \) and \( u(t) \) will be as follows,
\[
Tf_2 = \frac{-m_u s^2 - k_t}{\Delta}
\]

4. Simulation Results

In this section, firstly, SDRE based sliding mode controller applied to the quarter car model. The models parameters have following values [7].
Table 1. Parameters of quarter car model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_s )</td>
<td>504.5 kg</td>
</tr>
<tr>
<td>( m_u )</td>
<td>62 kg</td>
</tr>
<tr>
<td>( k_s )</td>
<td>13100 N/m</td>
</tr>
<tr>
<td>( c_s )</td>
<td>400 Ns/m</td>
</tr>
<tr>
<td>( k_t )</td>
<td>252000 N/m</td>
</tr>
</tbody>
</table>

The random input of disturbance is shown in Figure 2; the responses of suspension deflection, tyre deflection, sprung mass acceleration and force input are shown in Figure 3-6 respectively.

Figure 2. Disturbance Signal

Figure 3. Deflection of Suspension
In Figure 2 the random input is chosen randomly. For applications deflection of tire and deflection of suspension must be small values. According to the Figure 3 and Figure 4, it can be seen that the values are small in comparison to [7]. In Figure 5 the result of vertical acceleration caused by random input is shown. This value should also be small. If this value is not small displacement of car value will also not be small and that means big oscillations in vertical direction.
In Figure 6 the desired input force input result is seen. This force realised by a force generator. The $k$ value of SDRE based SMC is chosen 1.0 after some trials. In comparison to [7] this value is small and it means that the system is consumed less energy.

For now, PID control technique is applied. For PID control is chosen so control system scheme is as follows,
In Figure 8, the purple graph is road disturbance model and yellow line is the desired output for that disturbance and blue line is the displacement of car body. The PID constants are chosen 0.1, 0.5 and 5 after some trials.

5. Conclusion

In our study, an active vibration control of suspension system for quarter car body with two different controller. The first one is SDRE based sliding mode controller. In that controller, the sliding surfaces are designed by SDRE method. Then, these slopes are used for designing of sliding surfaces. The same model is also controlled by [7]. They use use both active, passive and semi active vibration control. But mainly they use MR damper for suspension system with $H_\infty$ control algorithm. In our study, the suspension system is give better results with consuming less energy. The k value in SDRE based SMC algorithm can be changed. If a better value for k is found the results will be better and more acceptable.

Lastly, the same system is controlled by classical PID. As seen in Figure 8. the desired value is not tracked accurately but acceptable.

References