Comparision of Three Vectoral Solutions for Kinematic Analysis of Mechanisms in Mechanical Engineering Education

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Abstract

A machine may contain one or more mechanism to fulfill its functionality. It is known fact that designing a mechanism is important aspect of machine design. Mechanism design usually consist of both kinematic and kinetic analysis. Kinematic analysis requires vector, scalar, and graphical analysis. The vector method, which is a preferable method, is mostly used in kinematic analysis. In this proposed study, three vector analysis methods will be studied in order to show their capabilities for the kinematic analysis of mechanisms. It is known that the vector polygon method is simple and requires a few analysis steps. However, it is not suitable for programming. The unit vector method seems to be easy but constructing of position, velocity, and acceleration vectors and calculating parameters are time consuming. Besides the analyst has to pay full attention when producing unit vectors. It is going to be proved that vector loop-closure method is the best systematic method. It is necessary that analyst should pay full attention about direction, sense, and derivatives of vectors.

Key words: Kinematic analysis, mechanism,vector loop-closure,vector polygon,unit vector.

1. Introduction

Mechanisms course has an important role for mechanical engineering education.This course includes position,velocity, and acceleration analysis as well as the another topics. These tasks are difficult for students, especially analysis of complex mechanisms. Students face some confuse about to get magnitude and direction of the vectoral quantities such as position,velocity, and aceleration. To overcome such confuses about applying of the methods in which solutions are made, there is a necessity to clear the advantages and disadvantages of the methods. In this study, the commonly used three solution methods has been explained with a simple example. Their capabilities has been introduced, the advantages and disadvantages criticized.

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2. The methods considered

In this study three vectoral methods are considered: The vector graph method, the unit vector method, and vector loop-closure method. To be able to show their advantages and disadvantages the best, a simle example is considered. Consider the simple example shown below to show how do the methods work.

Example problem

Consider the double slide mechanism in which, point A is moving to the right with a velocity of 2m/s and an acceleration of 1 m/s^2 when $x_A = 0.5$ m. Use the loop equation approach to determine the angular velocity of link AB. Link AB is 1 m long, and θ_3 is 120° in the position shown. Obtain the velocity and acceleration of the slider B.

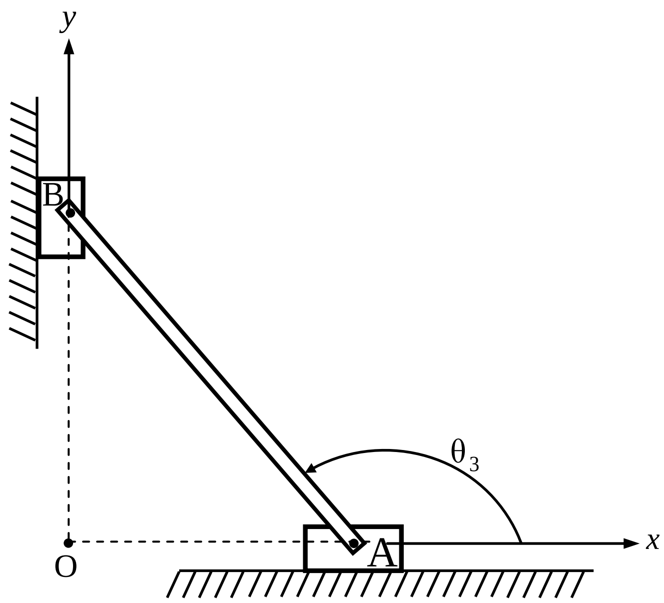


Figure 1. Double slide mechanism

2.1. The Vector Polygon Method

In this method, relative position,velocity,and acceleration relations are written between two point of a member. Then, they are added according to the vectoral summation rule. By adding the vectors, a triangle or polygon is obtained[1]. Unknowns can be found from this triangle or polygon by using any mathematical calculation such as Cosine, Sine theorem or resolving of a vector to its components.

2.1.1. Position Analysis

Apply the Sine theorem for the position polygon to get y_B ;

$$\frac{x_A}{\text{Sin}30^0} = \frac{y_B}{\text{Sin}60^0} \rightarrow \frac{0.5}{\text{Sin}30^0} = \frac{y_B}{\text{Sin}60^0} \rightarrow y_B = 0.87\,\text{m}$$

$$\vec{y}_B = \vec{x}_A + \vec{y}_{B/A} \;..............(1)$$

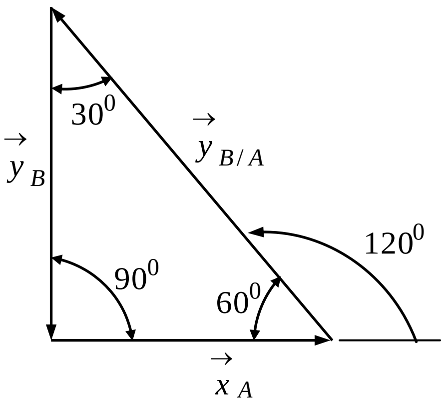


Figure 2. Position polygon

2.1.2. Velocity Analysis

Write down the relative velocity relation between point B, and A.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \;................(2)$$

At the instant shown, point A has a velocity of 2m/s to the right. The velocity of point B is not known,yet. But, we know its direction, which is vertical, from the motion of the mechanism. The relative velocity between B and A is not known, its direction must be perpendicular to the direction of AB.

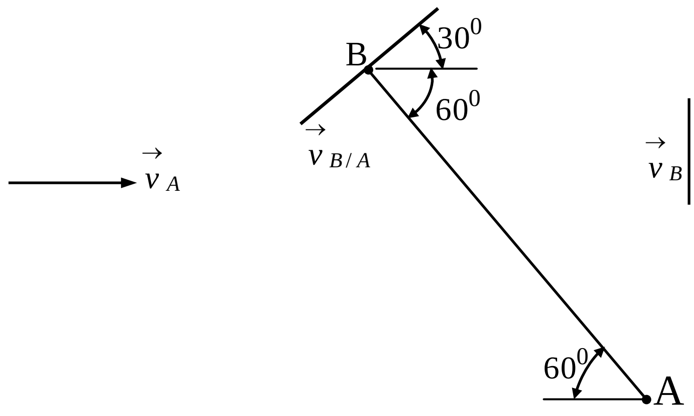


Figure 3. Directions of the velocities

A vectoral adding must be done following eqn(2) to get the velocity of point B.

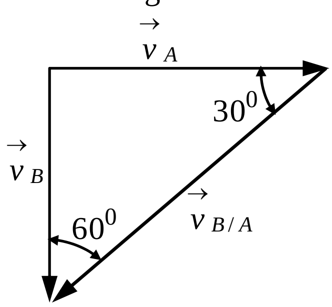
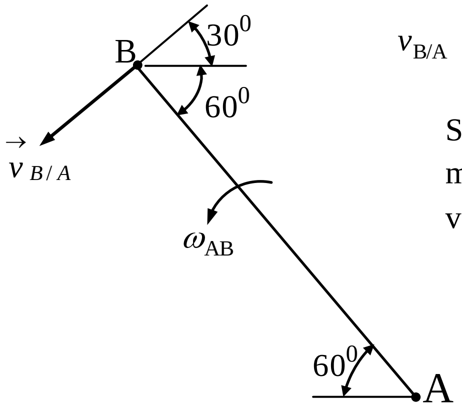


Figure 4. Velocity Polygon

From the Sine rule,it can be written as follows;

$$\frac{v_A}{\text{Sin}60^0} = \frac{v_B}{\text{Sin}30^0} = \frac{v_{B/A}}{\text{Sin}90^0} \Rightarrow \frac{2}{\text{Sin}60^0} = \frac{v_B}{\text{Sin}30^0} = \frac{v_{B/A}}{\text{Sin}90^0}$$

$v_B = 1.15\,\text{m/s}$ and $v_{B/A} = 2.31\,\text{m/s}$



$$v_{B/A} = \omega_{AB}|AB| \Rightarrow \omega_{AB} = \frac{2.31}{1} \Rightarrow \omega_{AB} = 2.31\,\text{rad/s (CCW)}$$

Since point B relatively rotates about point A, the sense of ω_{AB} must consistent with $v_{B/A}$. Thus, the sense of the angular velocity ω_{AB} is in CCW.

Figure 5. Determination of the direction of the angular velocity

2.1.3. Acceleration Analysis

As similar manner in velocity analysis, write down the relative velocity relation between point B, and A.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \text{.......................(3)}$$

At the instant shown, point A has an acceleration of $1 m/s^2$ to the right. The acceleration of point B is not known,yet. But, we know its direction, which is vertical, from the motion of the mechanism. The relative acceleration between B and A is not known, it has two components as normal (centripetal) ,and tangential acceleration. Since point B relatively rotates about point A, these acceleration components can be obtained as is rotation. The acceleration of point B can be found from the acceleration polygon according to eqn(4).

$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_n + (\vec{a}_{B/A})_t \text{.....................(4)}$$

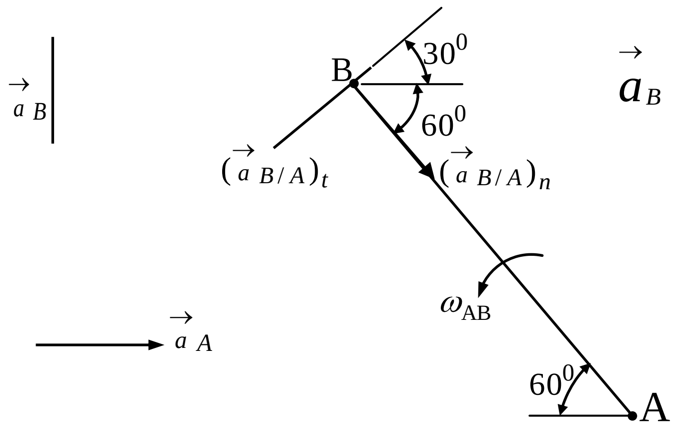


Figure 6. The accelerations of the mechanism

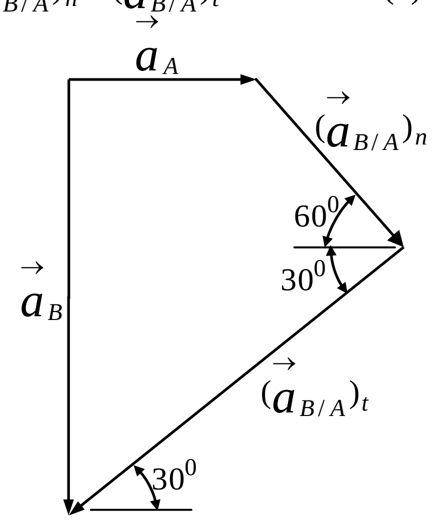


Figure 7. Accelerations polygon

$$(a_{B/A})_n = \omega_{AB}^2|AB| = 2.31^2 x1 = 5.34\,\text{m/s}^2$$

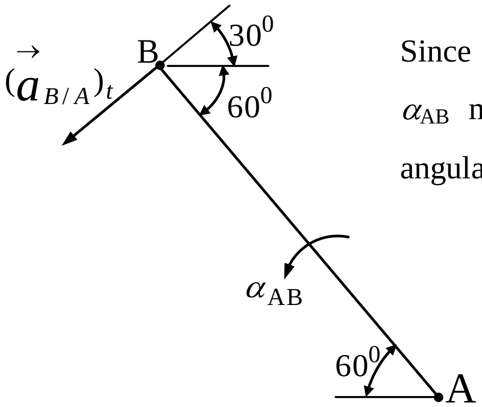
$$(a_{B/A})_t = \alpha_{AB}|AB| = \alpha_{AB}x1 = \alpha_{AB} \text{.....................(5)}$$

x Components

$$(\rightarrow +)\ 0 = a_{A} + (a_{B/A})_n Cos(60^0) - (a_{B/A})_t Cos(30^0)$$

$$(\rightarrow +)\ 0 = 1 + 5.34 x Cos(60^0) - (a_{B/A})_t Cos(30^0) \ \rightarrow \ 0 = 1 + 5.34 x Cos(60^0) - (a_{B/A})_t Cos(30^0)$$

$(a_{B/A})_t = 4.24\,\text{m/s}^2$ Substituting it into eqn(5), $4.24 = \alpha_{AB} x1 \Rightarrow \alpha_{AB} = 4.24\,\text{rad/s}^2$ ....(6)



Since point B relatively rotates about point A, the sense of α_{AB} must consistent with $(a_{B/A})_t$. Thus, the sense of the angular velocity α_{AB} is in CCW.

Figure 8 The defining the direction of the angular acceleration

y Components

$$(\uparrow +)\ -a_{B} = -(a_{B/A})_n Sin(60^0) - (a_{B/A})_t Sin(30^0)$$

$$(\uparrow +)\ -a_{B} = -5.34 Sin(60^0) - 4.24 Sin(30^0) \ \Rightarrow \ a_{B} = 6.74\,\text{m/s}^2 (\downarrow)$$

2.2. The Unit Vector Method

This method uses relative position, velocity,and acceleration relations between two points[2]. All vectors such as position,velocity, and acceleration are defined in terms of unit vectors, *i*,*j*, and *k*. Where, *i*, *j*, and *k* stand for the unit vectors for *x*,*y*,and *z* coordinates,respectively. All calculations are done according to vector adding, substracting, and producing rules.

2.2.1 Velocity Analysis

Relating velocities of point A and B,it gives as follow.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \ ....(2)$$

$$(v_{B/A}) = \vec{\omega}_{AB}\ x\ \vec{r}_{B/A} \ ....(8) \qquad (v_{B/A}) = \left[-|AB| Cos(60^0)\,\vec{i} + |AB| Sin(60^0)\,\vec{j}\right] x\, \omega_{AB}\,\vec{k}$$

$$(v_{B/A}) = \omega_{AB}\,\vec{k}\, x \left[-1 Cos(60^0)\,\vec{i} + 1 Sin(60^0)\,\vec{j}\right] x \ \Rightarrow (v_{B/A}) = -0.5\omega_{AB}\,\vec{j} - 0.87\omega_{AB}\,\vec{i}$$

If $v_{B/A}$ is substituted into eqn(2), it becomes as $v_B \vec{j} = 2\vec{i} - 0.5\omega_{AB}\vec{j} - 0.87\omega_{AB}\vec{i}$ ....(9)

Equate the both sides of eqn(9);

$$0 = 2 - 0.87\omega_{AB} \Rightarrow \omega_{AB} = 2.3\,\text{rad/s (CCW)}$$

$$v_B = -0.5\omega_{AB} \Rightarrow v_B = -0.5(2.3) \Rightarrow v_B = -1.15\,\text{m/s}\,(\downarrow)$$

2.2.2. Acceleration Analysis

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}\ \text{................(10)} \qquad \vec{a}_{B/A} = \vec{\alpha}\,x\,\vec{r}_{B/A} + \vec{\omega}_{AB}\,x(\vec{\omega}_{AB}\,.x\,\vec{r}_{B/A})\ \text{............(11)}$$

Combining eqn(10) and eqn(11),it becomes as follow

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}\,x\,\vec{r}_{B/A} + \vec{\omega}_{AB}\,x(\vec{\omega}_{AB}\,.x\,\vec{r}_{B/A})\ \text{............(12)}$$

Assuming that member AB rotates in CCW direction at this instant,namely, $\vec{\alpha} = \alpha\vec{k}$

By replacing all terms into eqn(12),

$$a_B\vec{j} = a_A\vec{i} + \alpha\vec{k}\,x\left[-|AB|Cos(\theta)\vec{i} + |AB|Sin(\theta)\vec{j}\right] + \omega_{AB}\vec{k}\,x\left\{\omega_{AB}\vec{k}\,.x\left[-|AB|Cos(\theta)\vec{i} + |AB|Sin(\theta)\vec{j}\right]\right\}\text{............(13)}$$

$$a_B\vec{j} = 1\vec{i} + \alpha\vec{k}\,x\left[-1Cos(60^0)\vec{i} + 1Sin(60^0)\vec{j}\right] + 2.31\vec{k}\,x\left\{2.31\vec{k}\,.x\left[-1Cos(60^0)\vec{i} + 1Sin(60^0)\vec{j}\right]\right\}\text{............(14)}$$

$$a_B\vec{j} = 1\vec{i} + \left(-0.5\alpha\vec{j} - 0.87\alpha\vec{i}\right) + 2.31\vec{k}\,x\left(-1.155\vec{j} - 2\vec{i}\right)$$

$$a_B\vec{j} = 1\vec{i} + \left(-0.5\alpha\vec{j} - 0.87\alpha\vec{i}\right) + 2.67\vec{i} - 4.62\vec{j} \Rightarrow a_B\vec{j} = (1 - 0.87\alpha + 2.67)\vec{i} + (-0.5\alpha - 4.62)\vec{j}\ \text{....................(15)}$$

Equate the both sides of eqn(15) to get the angular acceleration of the rod and rectilinear acceleration of point B;

$$(1 - 0.87\alpha + 2.67) = 0 \Rightarrow \alpha = 4.22\,rad/s^2$$

$$a_B = -0.5\alpha - 4.62 \Rightarrow a_B = -0.5(4.22) - 4.62 \Rightarrow a_B = -6.73\,\text{m/s}^2\,(\downarrow)$$

2.3. Vector Loop -Closure Method

The vector loop-closure method deals with modelling of a mechanism in terms of complex numbers method. A loop equation is generated for every closed-loop in terms of vectors. This equation is resolved real and imaginary part according to complex number theory[3].Thus, two equations are obtained for every loop. It is well known that in the vector loop-closure, one must construct the vector polygon first. To construct it, members and other auxiliary lines are defined by vectors as shown in Fig 9.a so that they form a closed-loop. Every vector must be defined by an angle which shall be determined from horizontal to the head of vector in direction of CCW as shown in Fig 9.a and 9.b. After construction of the vector loop-closure, a rotation direction (either CW or CCW) must be assumed as positive direction to write down vector loop-closure equation as is in eqn.(1). Once position vector is obtained then velocity and acceleration vectors are easily obtained by taking the first and second derivative of position vector with respect to time, respectively. In the method, CCW direction is positive for rotation.

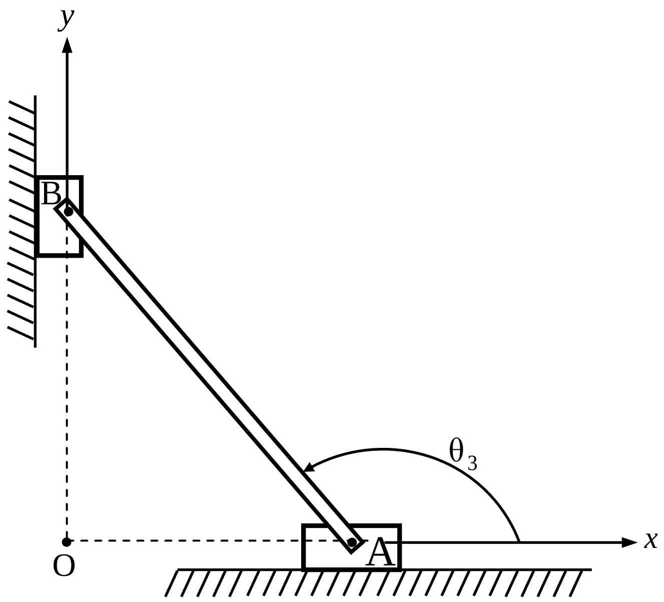


Figure 1 Double slide mechanism

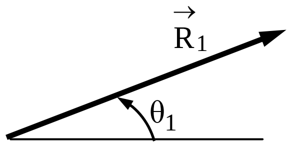


Figure 9.b. Defining the direction of the angle

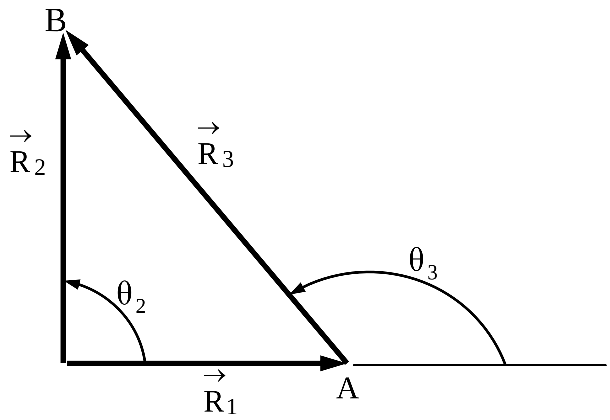


Figure 9.a. The position vector loop

Assuming that CCW rotation in the vector loop as shown in Figure 9.b is positive, the vector loop-closure is written as follow.

$$\vec{R}_1 - \vec{R}_2 + \vec{R}_3 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Writing it in Euler's identity form we have

$$x_A e^{i\theta_1} - y_B e^{i\theta_2} + l e^{i\theta 3} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Noticing that $e^{i\theta} = Cos\theta + iSin\theta$ and expanding it in eqn(17),it becomes as in eqn(18).

$$x_A(Cos\theta_1 + iSin\theta_1) - y_B(Cos\theta_2 + iSin\theta_2) + l(Cos\theta_3 + iSin\theta_3) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

2.3.1. The position Analysis

The Real Part of eqn(18) is:

$$x_A Cos\theta_1 - y_B Cos\theta_2 + lCos\theta_3 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

The Imaginary Part of eqn(18) is:

$$x_A Sin\theta_1 - y_B Sin\theta_2 + lSin\theta_3 = 0 ....(20)$$

l, θ_1, and θ_2 are constant; x_A, and θ_3 are variable

Substituting the values l=1 m, x_A=0.5 m, θ_1= 0^0, θ_2= 90^0, θ_3=120^0 into the eqn.(20)

$$0.5Sin(0^0) - y_B Sin(90^0) + 1Sin(120^0) = 0 ....(21)$$

It gives y_B=0.87 m.

2.3.2. The Velocity Analysis

The velocity equations are obtained by taking the first derivative of position equations (18) and (19) with respect to time, respectively.

$$\dot{x}_A Cos\theta_1 - \dot{y}_B Cos\theta_2 - l\dot{\theta}_3 Sin\theta_3 = 0 ....(22)$$

$$\dot{x}_A Sin\theta_1 - \dot{y}_B Sin\theta_2 + l\dot{\theta}_3 Cos\theta_3 = 0 ....(23)$$

Noticing that $\dot{x}_A = v_A ; \dot{y}_B = v_B$ and $\dot{\theta}_3 = \omega_{AB}$

Rearranging eqn(22) and (23),they becomes as

$$v_A Cos\theta_1 - v_B Cos\theta_2 - l\omega_{AB} Sin\theta_3 = 0 ....(24)$$

$$v_A Sin\theta_1 - v_B Sin\theta_2 + l\omega_{AB} Cos\theta_3 = 0 ....(25)$$

$$2Cos0^0 - v_B Cos90^0 - 1\omega_{AB} Sin(120^0) = 0 ....(26)$$

$0.87\omega_{AB} = 2 \rightarrow \omega_{AB} = 2.3$ rad/s

Substitute ω_3 into eqn (25) to get v_B

$$2Sin0^0 - v_B Sin90^0 + 1x2.3Cos(120^0) = 0$$

$v_B = -1.15$ m/s.

Where, negative sign means that the slider B has a downward velocity.

2.3.3. The Acceleration Analysis

The acceleration equations are obtained as follows by taking the first derivative of velocity equations (22) and (23) with respect to time,respectively.

$$\ddot{x}_A Cos\theta_1 - \ddot{y}_B Cos\theta_2 - l\ddot{\theta}_3 Sin\theta_3 - l(\dot{\theta}_3)^2 Cos\theta_3 = 0 ....(27)$$

$$\ddot{x}_A Sin\theta_1 - \ddot{y}_B Sin\theta_2 + l\ddot{\theta}_3 Cos\theta_3 - l(\dot{\theta}_3)^2 Sin\theta_3 = 0 ....(28)$$

Noticing that $\ddot{x}_A = a_A; \ddot{y}_B = a_B$ and $\ddot{\theta}_3 = \alpha_{AB}$

Rearranging eqn(27) and (28),they becomes as

$$a_A Cos\theta_1 - a_B Cos\theta_2 - l\alpha_{AB} Sin\theta_3 - l(\omega_{AB})^2 Cos\theta_3 = 0 \qquad (29)$$

$$a_A Sin\theta_1 - a_B Sin\theta_2 + l\alpha_{AB} Cos\theta_3 - l(\omega_{AB})^2 Sin\theta_3 = 0 \qquad (30)$$

Substitute the numerical values into eqn.(29) to get the angular acceleration

$$1xCos0^0 - a_B Cos90^0 - 1x\alpha_{AB} Sin(120^0) - 1(2.3)^2 Cos(120^0) = 0 \qquad (31)$$

$$1 - 0.87\alpha_{AB} + 2.645 = 0 \rightarrow \alpha_{AB} \cong 4.2 \text{ rad/s}^2$$

Where, positive sign means that the rod has an angular acceleration in direction of CCW.

Substitute α_{AB} into eqn (29) to get the acceleration of point B

$$1Sin(0^0) - a_B Sin(90^0) + 1x4.2Cos(120^0) - 1x2.3^2 Sin(120^0) = 0$$

$$-a_B - 2.1 - 4.58 = 0 \rightarrow a_B = -6.68 \text{ m/s}^2$$

Where, negative sign means that the slider B has a downward acceleration.

3. Results and Discussion

Three methods generated the same results as shown in Table 1.The differences between the results are due to rounding the digits up.

Table 1. The results obtained from the three methods

The Methods	ω_{AB} (rad/s)	v_B (m/s)	$\alpha_{AB}(rad/s^2)$	$a_B(m/s^2)$
Vector polygon method	2.31	1.15	4.24	6.74
Unit Vector Method	2.3	1.15	4.22	6.73
Vector Loop-Closure Method	2.3	1.15	4.2	6.68

Although *the vector polygon method* is simple and easy to achieve operations, analyst can able to see which direction shall a vector lie down. This is important to define direction of velocity or acceleration. After polygon solution, analys must be define the direction of angular velocity and angular acceleration according to obtained results from the polygon. Solution by this method is only valid for instant considered. After this instant the analyst must be solve problem again. Although this method is very simple,it is not suitable for programming.This is the biggest problem for this method. *The unit vector method* reuired the minimum operation. But it consumed much time because, operations between the unit vectors require to be very careful. This method automatically generates direction of velocity and acceleration according to the defined coordinates. But this method is not suitable for programming,too. *The vector loop-closure method* is based on construction of vector loop and getting the position equation. After the position equation,velocity and acceleration equations are obtained by deriving the position vector. The full attention shall pay for deriving of equations. This method is a systematic method. Once analyst established the solution steps,it can be used all instants of mechanism. Because of its this property, it is suitable for programming. In this method direction of velocity and acceleration is automatically generated according to the defined coordinates. It is concluded that someone who needs only to analyze for the one instant of mechanism he/she is advised to use *the vector polygon method*. If the analyst needs to analyze for the sequent instant of mechanism, he/she is advised to use *the vector loop-closure method.*

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