Chaos Control of the Chaotic Symmetric Gyroscope System

*[1]Barış CEVHER, [2]Yılmaz UYAROĞLU and [3]Selçuk EMIROĞLU
[1,2,3] Faculty of Engineering, Department of Electrical and Electronics Engineering
Sakarya University, Turkey

Abstract: In this paper, the chaos control for the Gyroscope System is performed by using control methods based on sliding mode control and time-delay feedback control. The designed controllers for the complete chaos control of Gyroscope System are obtained using sliding mode control theory, Lyapunov stability theory and time-delayed feedback control theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve chaos control of similar systems. In spite of its inherent limitations, Time Delayed Feedback Control can be applied successfully in many chaos control applications. So chaos control of the Gyroscope System is ensured completely by time delayed feedback control. Numerical simulations are presented to demonstrate the effectiveness and validate the chaos control of Gyroscope Systems.

Key words: Gyroscope System, Chaos Control, Time-Delayed Feedback Control, Sliding Mode Control

1. Introduction

In 1963, Lorenz found the first chaotic attractor, which is named as Lorenz chaotic system, in a three dimensional autonomous system when he studied atmospheric convection [1]. After Lorenz, chaos has been extensively interesting study area for many scientists and also, many chaotic systems were introduced such as Rössler system [2], Chen system [3], Lü system [4], Gyroscope System [5].

However, when chaotic behavior is sometimes undesirable, the chaotic behavior of system should be controlled. So, chaos control has become one of the much interesting research subject, since the control of chaotic systems is firstly proposed by Ott, Grebogi and Yorke [6]. Recently, many control methods are proposed for the control of the chaotic systems. The control strategies applied to control of chaos such as OGY method [6], adaptive control [7], passive control [8, 9, 10], delayed feedback control [11, 12], backstepping control [13], sliding mode control [14, 15].

Chen investigated the chaotic behavior of two dimensional gyroscope system in 2002, which is called the gyro system [5]. In this paper, chaos in gyro system is controlled to zero equilibrium point by using delay feedback and sliding mode control. Based on delay feedback and sliding mode control theory, we prove that the designed controller can control gyro system to the zero equilibrium point. Numerical simulations are given for illustration and verification.

This paper is organized as follows. Section 2 briefly introduced the mathematical model of gyroscope system. Sliding mode control design principles and time delay feedback control theory are presented in Section 3. The controller is designed to control gyro chaotic system based on time delay and sliding mode control method and numerical simulations for chaos control is given in section 4. In section 5, conclusions are finally given.

2. Mathematical model of Gyroscope System

The chaotic dynamics and bifurcation diagram of this symmetric gyro system are extensively studied in ref [5, 16]. The gyroscope system (called gyro) is given by:

$$\ddot{\theta} + \alpha^2 \frac{(1-cos\theta)^2}{sin^3\theta} - \beta_1 sin\theta + c_1\dot{\theta} + c_2\dot{\theta}^3 = f_1 sin\omega_1 t sin\theta \ . \tag{1}$$

For simplicity, $x_1=\theta,\ x_2=\dot{\theta}$ notations are introduced. By using these notations, the equation governing the motion of the gyro after necessary transformation is given by as follow:

$$\left.\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \alpha^2 \frac{(1-cosx_1)^2}{sin^3x_1} - c_1x_2 - c_2x_2^3 - \beta_1 sinx_1 + f_1 sin\omega_1 t sinx_1 \end{aligned}\right\} \tag{2}$$

3. Sliding Mode Control Design For Chaos Control of in Gyroscope System

Suggested Gyroscope System chaotic system is described in equation (4). Thus the controlled chaotic system of Gyroscope System is attained as follows:

$$\left.\begin{aligned} \dot{x_1} &= x_2 + u_1 \\ \dot{x_2} &= \alpha^2 \frac{(1-cosx_1)^2}{sin^3x_1} - c_1x_2 - c_2x_2^3 - \beta_1 sinx_1 + f_1 sin\omega_1 t sinx_1 + u_2 \end{aligned}\right\} \tag{3}$$

where u_1, u_2, are control signals.

$$e = x - x_d \tag{4}$$

where e $=[e_1\ e_2]^T$ is the tracking error vector. The error dynamics may be written as below:

$$\dot{e} = \dot{x} - \dot{x}_d = Ax + Bg + Bu - \dot{x}_d \tag{5}$$

where A is the system matrix, B is the control matrix, and g represents the system nonlinearities plus parametric uncertainties in the system. The control problem is to get the state x= $[x_1\ x_2]^T$ to track a specific time varying state $x_d = [xd_1\ xd_2]^T$ in the presence of nonlinearities.

$A = \begin{bmatrix} 0 & 1 \\ 0 & -c_1 \end{bmatrix}$; $B=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $g= \begin{bmatrix} 0 \\ G \end{bmatrix}$; $G = \alpha^2 \frac{(1-cosx_1)^2}{sin^3x_1} - c_2x_2^3 - \beta_1 sinx_1 + f_1 sin\omega_1 t sinx_1$

Now, a time varying proportional plus integral (PI) sliding surface s (e, t) ∈ R2 is defined by the scalar equation s = s (e, t) as

$$s = Ke - \int_0^t K(A - BL)e(\tau)d\tau \quad (6)$$

where K∈R3x3, which must satisfy det(KB) ≠ 0, is a gain matrix, and L ∈ R2x2, which must have a stable A-BL, is a gain matrix, namely, the eigenvalues λ_i (i=1,2,3) of the matrix A-BL are negative (λ_i |< 0). It is well known that when the system operates in the sliding mode, the sliding surface and its derivative must satisfy s = ṡ = 0 [17, 18]. The equations may be written as below:

$$\dot{s} = KBg + KBLe + KBu + KAx_d - K\dot{x}_d = 0 \quad (7)$$

Since KB is non-singular, the equivalent control in the sliding mode is given by

$$u_{eq} = -[\, \hat{g} + Le \,] - (KB)^{-1} [\, KAx_d - K\dot{x}_d \,] \quad (8)$$

where g is not exactly known, but guessed as ĝ, and the estimation error on g is presumed to be restricted by some known function G such that ‖ g - ĝ ‖ ≤ G. In addition, it reveals that the stability of systems in the sliding motion can be guaranteed just by selecting an appropriate matrix L using any pole assignment method. To ensure the achievement of the reaching condition indicated in equation (7), a control law is proposed as:

$$u = u_{eq} - (KB)^{-1}[\varepsilon + \| KBG \|]sign(s) \quad (9)$$

where $\varepsilon > 0$.

4. Time-Delayed Feedback Control Method For Chaos Control of in Gyroscope System

Pyragas (1992) showed that chaotic behavior could be controlled by using time delayed feedback control method [19]. The control of chaotic Gyroscope System is achieved using time delay feedback control theory. The controlled Gyroscope System model given by

$$\left.\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \alpha^2 \frac{(1-cosx_1)^2}{sin^3 x_1} - c_1 x_2 - c_2 x_2^3 - \beta_1 sinx_1 + f_1 sin\omega_1 t sinx_1 + u \end{aligned}\right\} \quad (10)$$

The controller $u(t)$ is designed based on time delay feedback control[**] as in Eq.

$$u(t) = K_c[x_2(t) - x_2(t-\tau)] \quad (11)$$

$u(t)$ obtained that the difference between current value of system variable $x_2(t)$ and its τ seconds previous multiplied by constant K_c, where K_c is feedback gain.

5. Numerical Simulations for Chaos Control of in Gyroscope System

In this section, the gyroscope system is controlled to a chaotic orbit by a Sliding mode control and time-delayed feedback control. Numerical simulations are applied to confirm the effective and the feasible of the proposed control methods.

5.1 Applied sliding mode control method to the Gyro system

Equation (4) is substituted with the numerical values as follows:

$$\begin{aligned} \dot{x_1} &= x_2 + u_1 \\ \dot{x_2} &= \alpha^2 \frac{(1-cosx_1)^2}{sin^3 x_1} - c_1 x_2 - c_2 x_2^3 - \beta_1 sinx_1 + f_1 sin\omega_1 t sinx_1 + u_2 \end{aligned} \tag{12}$$

Where α=10, c_1=1, c_2=0.05, β_1=1, f_1=35.5, ω_1=2

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix};\ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\quad g = \begin{bmatrix} 0 \\ G \end{bmatrix}$$

Where $G = \alpha^2 \frac{(1-cosx_1)^2}{sin^3 x_1} - c_2 x_2^3 - \beta_1 sinx_1 + f_1 sin\omega_1 t sinx_1$

Here, the gain matrix K is chosen as K = diag (1, 1) such that KB = diag (1, 1) is nonsingular. The desired eigenvalues of the matrix A-BL are taken as P = [-5 -5.001]. The gain matrix L is found as follows by using the pole placement method:

$$L = \begin{bmatrix} 5 & 1 \\ 0 & 4.001 \end{bmatrix}.$$

As a result, the matrix K(A-BL) is computed as K(A-BL) = diag (-5, -5.001). The PI switching surfaces are obtained as follows:

$$\left.\begin{aligned} s_1 &= e_1 + \int_0^t 5e_1(\tau)d\tau \\ s_2 &= e_2 + \int_0^t 5.001 e_2(\tau)d\tau \end{aligned}\right\} \tag{13}$$

For this numerical simulation, the initial points of the system are employed as [$x_1(0)$, $x_2(0)$] = [-0.2, 0.1]. The constant controller coefficient ε is selected as ε<0.5. The reference states xd_1, xd_2, xd_3 are selected as $xd_1 = xd_2 = x_d$. Therefore, the control signals may be attained as:

$$\left.\begin{aligned} u_1 &= [\dot{x}_d - e_2 + 5e_1 - x_d + x_2x_3 - \varepsilon\,\mathrm{sign}(s_1)] \\ u_2 &= \Big[-4.001e_2 + x_d + \dot{x}_d + \sin(x_1) + \frac{100(1-cosx_1)^2}{sin^3x_1} - 35.5sin2tsinx_1 \\ &\quad -\mathrm{sign}(s_2)\left(\varepsilon + \left|\sin(x_1) - \frac{100(1-cosx_1)^2}{sin^3x_1} - 35.5sin2tsinx_1 + \frac{x_2^3}{20}\right|\right) + \frac{x_2^3}{20}\Big] \end{aligned}\right\} \quad (14)$$

5.2 Applied Time-Delayed Feedback Control method to the Gyro system

The equation (10) which has been added the controller is expressed with the numerical values as follows:

$$\left.\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \alpha^2\frac{(1-cosx_1)^2}{sin^3x_1} - c_1x_2 - c_2x_2^3 - \beta_1 sinx_1 + f_1 sin\omega_1 t sinx_1 + u \end{aligned}\right\} \quad (15)$$

Where α=10, c_1=1, c_2=0.05, β_1=1, f_1=35.5, ω_1=2

The controller *u(t)* is designed based on time delay feedback control [19] as in Eq.

$$u(t) = K_c[x_2(t) - x_2(t-\tau)] \quad (16)$$

Where $K_c = 4$; τ =0.5

According to numerical simulations, time series of Gyro system have been obtained as respectively shown in Figure 1; phase portrait and time series of without controlled Gyro system, in Figure 2; controlled Gyro system with SMC, in Figure 3; controlled Gyro system with time-delayed feedback control, in Figure 4; applied the control signals to the Gyro system after 10s.

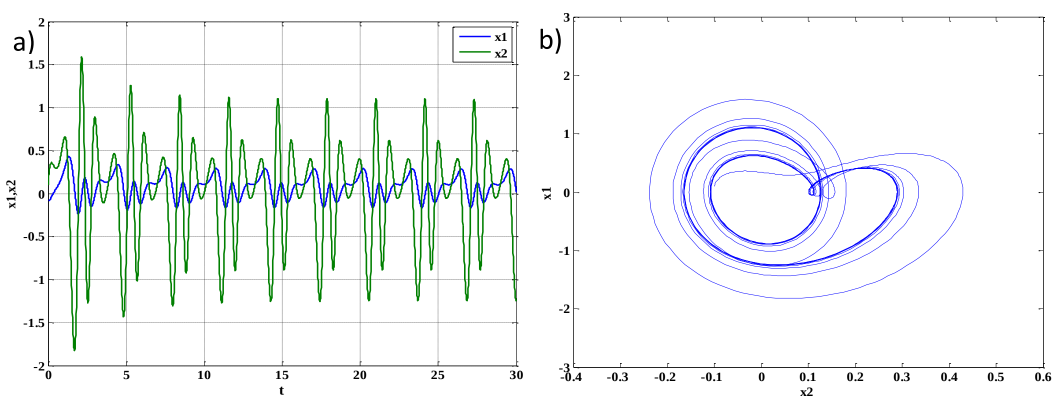

Figure 1. Time series of without controlled gyro system in x1-x2-t plane b) phase portrait of gyro system

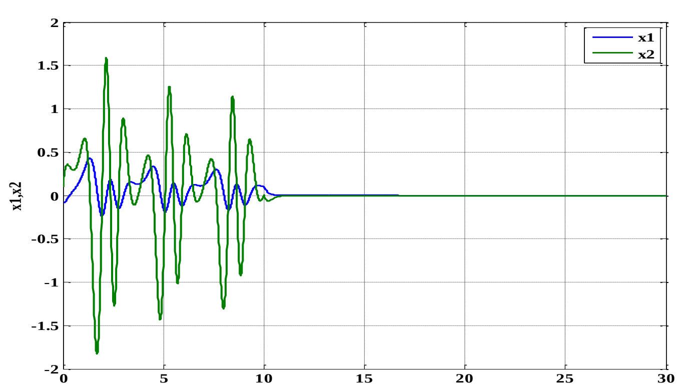

Figure 2. Time series of controlled gyro system in x1-x2-t plane with SMC after 10s

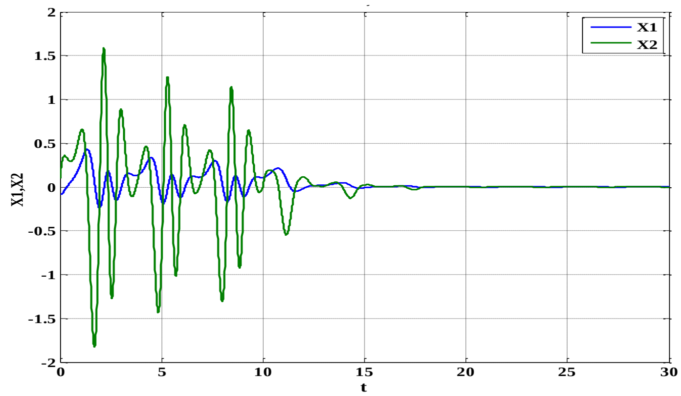

Figure 3. Time series of controlled gyro system in x1-x2-t plane with time-delayed feedback control after 10s

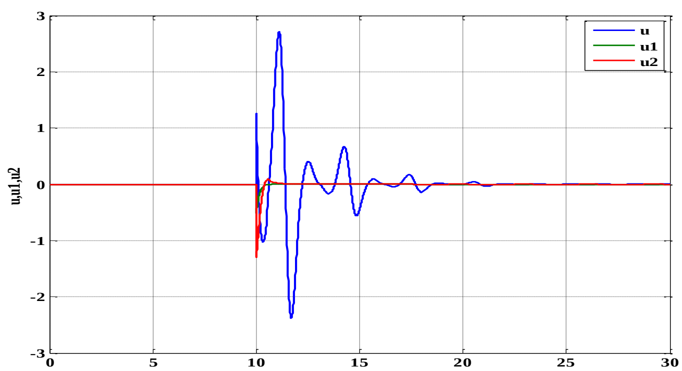

Figure 4. Applied the control signals to the gyro system after 8s

6. Conclusions

In this paper, effective control techniques have been suggested to stabilize chaos Gyroscope chaotic system. A sliding mode control law is applied by using a PI switching surface. So, it is found the stability of the error dynamics in the sliding mode that easily ensured by the PI switching surface. Designed SMC controller is rather satisfactory to a nonlinear controller to eliminate the undesirable chaotic oscillations. Several simulations results are presented. The simulation results indicate that the proposed control scheme works well. The control scheme was able to stabilize the chaotic Gyroscope System around user-defined set-points. In addition, the control was able to induce chaos on the stable Gyroscope System. In this paper, proposed S.M. Controller can be performed in similar systems. Time-delayed feedback control can be easily implemented to control periodic orbits in complex dynamical systems. Therefore, this time-delayed feedback control is very useful. Finally, numerical simulations are provided to show the effectiveness of proposed methods. The reaching results are satisfied in view of.

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